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Applying graph multiplication method to solve the bar systems in strenghth of materials

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ABSTRACT: The energy method is a powerful tool, used in many scientific disciplines related to material deformation of structures such as in mechanical engineering, construction, and bridges. Vereshchagin method, or graph multiplication method, developed based on the energy transformation principle of deformation solid mechanics, is applied to solve the problems of deflection or determining vertical and rotational displacement and strains of body deformation. However, most of research work early published focus only on beam problems. There are few studies dealing with the use of this method for solving bar systems. In this paper, the problems related to bar systems are solved using the Vereshchagin method through specific examples. The results in the paper can be considered as an important guide in industry and can be used in teaching.

KEYWORDS: Vereshchagin method, strength of materials, stress, external force, displacement.

I. INTRODUCTION

Vereshchagin method, representative by graph multiplication method, shows the effective ability to determine displacement resulting from deformation body of bending structures. The advantages of this method is the integrating process derived from Maxwell-Mohr [1-4]. Thanks to Vereshchagin method, the origin problem can be changed to elementary algebraic procedure on two bending moment diagram in the actual and unit states. Many studies dealing with Vereshchagin method have focused on beam problem, but there are few studies related to bar system or tensile or compressive problems [5-8]. This article presents the specific application in bar systems of mechanics of solid which has not yet documented in any available research work.

II. FUNDAMENTAL OF GRAPH MULTIPLICATION METHOD

Similar to the derivation of graph multiplication method for bending problem which is presented in [9]. Those of bar problems can be denoted as N_P is the internal force diagram resulting from external force in axial direction. $\overline{N_k}$ is the internal force diagram generated by unit state $P_k = 1$ ($M_k = 1$), put at the position where the displacement (or rotational displacement) is to be calculated.

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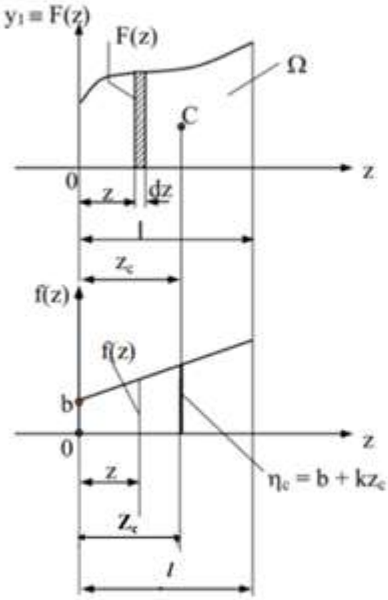

Fig -1: Schematic graph showing the derivation of multiplication method.

The general formula for determining vertical and rotational displacement in the linear elastic system as below:

$$\Delta_{km} = \sum\int \frac{\overline{N_k} N_m dz}{EF} + \sum\int \frac{\overline{M_k} M_m dz}{EJ} + \sum\int \eta \frac{\overline{Q_k} Q_m dz}{GF}$$

(1)

Two function of F(z) and f(z) simultaneously vary in the interval [0, l] and satisfy the integration as

$$I = \int_0^t F(z).\mathrm{f(z)}\,\mathrm{dz} \quad (2)$$

If two functions F(z) and f(z) satisfy 3 conditions.

1- F(z), f(z) is continuous in the interval [0, l].

2- F'(z), f'(z) are continuous in the interval [0, l].

3- f(z) is a first order function (or less than first order) of the form: f(z) = b + k (z).

Then the integral I will be calculated according to the formula:

$I = F.f = \Omega.\eta c$ (3)In which:

F is the graph below the function F(z).

f is the graph below the function f(z). Ω is the area of the graph under the function F(z) with center C (Figure 1). η_c is the coordinate corresponding to the centroid of the area and taken on the graph below the function f(z)

From (1) we deduce: if the internal force diagrams M_m , N_m , Q_m caused by the load have any form, and the graphs, and $\overline{M_m}, \overline{N_m}, \overline{Q_m}$ resulting from the unit state is in the form of a straight line (first order or constant), then:

$$\Delta_{km} = \sum \frac{1}{EF}\Omega(N_m)\overline{N_k}(c) + \sum \frac{1}{EJ}\Omega(M_m)\overline{M_k}(c) + \sum \frac{1}{GF}\Omega(Q_m)\overline{Q_k}(c) \quad (2)$$

Where:

$\Omega(N_m), \Omega(M_m), \Omega(Q_m)$ are the area of internal diagram of M_m, N_m, Q_m respectively.

$\overline{N_k}(c), \overline{M_k}(c), \overline{Q_k}(c)$ are the values of $\overline{M_m}, \overline{N_m}, \overline{Q_m}$ at positions corresponding to the centroids of the areas of the internal diagram of M_m, N_m, Q_m respectively.

From (2) we find that if on a bar or beam there is only N_m ($M_m = 0$, $Q_m = 0$), this is the case where the tensile (compression) force is at the center. In order to clearly make the above theory, some specific examples will be presented in next part. The application of the method mentioned above to solve the problem of the center of tension (compression) bar system by the graph multiplication method.

III. SOME APPLICATION EXAMPLES

Example 1: For the bearing bar system as shown in Figure 2. Given that bars 1, 2, 3 have the same material (E) length l = a and circular cross-section with area F. The allowable stress is [σ].

Determine:

1. Internal force in bars
2. Displacement at point C, rotational displacement of the section at B

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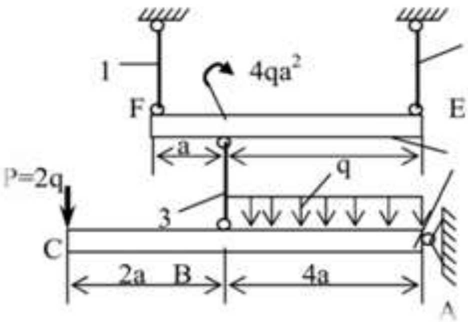

Fig- 2: Structural information for Example 1.

Solution

1. Determining the internal forces in the bars using the static equilibrium equations, it can be solved the internal forces in the bars as follows:

$$N_1 = \frac{16qa}{5},\ N_2 = \frac{9qa}{5},\ N_3 = 5qa$$

2. Identifying displacement at C and rotational displacement at B:

* The displacement at C

Put the Pk = 1 at C and solving the internal for in this case. It is as the follow

$$N_1 = \frac{6}{5},\ N_2 = \frac{3}{10},\ N_3 = \frac{3}{2}$$

The internal force diagram resulting from unit state $\overline{N_k}$ (as presented in Figure 3b)

The internal force diagram for actual state N_P (as presented in Figure 3c).

$$f_c = \frac{1}{EF}\Omega_i\eta_i = \frac{1}{EF}\left[\Omega_1\eta_1 + \Omega_2\eta_2 + \Omega_3\eta_3\right]$$

Where:

$$\Omega_1 = \frac{16qa}{5}.a = \frac{16qa^2}{5}, \eta_1 = \frac{6}{5},$$

$$\Omega_2 = \frac{9qa}{5}.a = \frac{9qa^2}{5}, \eta_2 = \frac{3}{10},$$

$$\Omega_3 = 5qa.a = 5qa^2, \eta_3 = \frac{3}{2}$$

$$\Rightarrow f_c = \frac{1}{EF}\left[\frac{16qa^2}{5}.\frac{6}{5} + \frac{9qa^2}{5}.\frac{3}{10} + 5qa^2.\frac{3}{2}\right] = \frac{297qa^2}{25}$$

* Determining rotational displacement at B:

Put the unit moment Mk = 1 as Figure 4. The results are solved as:

$$\overline{N_1} = \frac{1}{5a},\ \overline{N_2} = \frac{1}{20a},\ \overline{N_3} = \frac{1}{4a}$$

The values of these Ω_i as the case for calculating the displacement.

$$\Rightarrow \theta_c = \frac{1}{EF}\left[\frac{16qa^2}{5}.\frac{1}{5a} + \frac{9qa^2}{5}.\frac{1}{20a} + 5qa^2.\frac{1}{4a}\right] = \frac{99qa^2}{50}$$

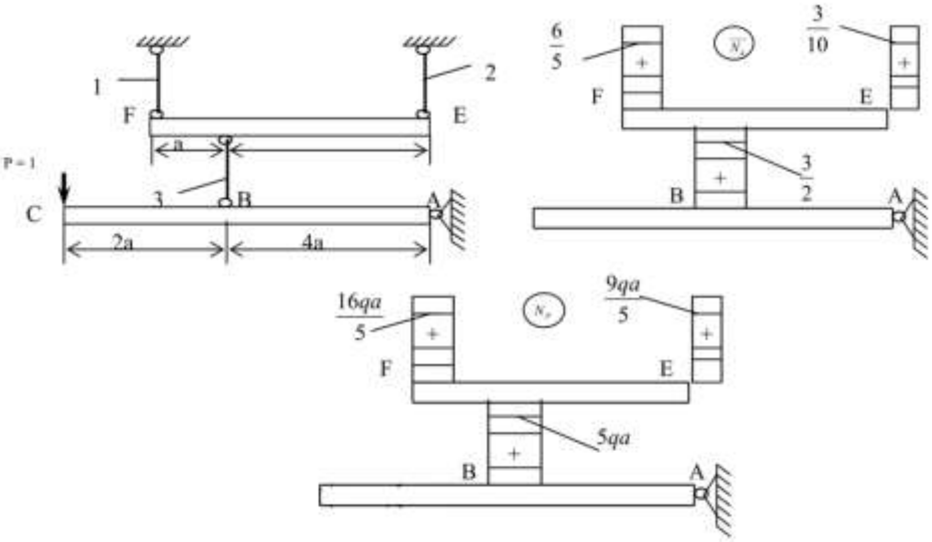

Fig -3: Internal diagram given by actual state and unit state for Example 1.

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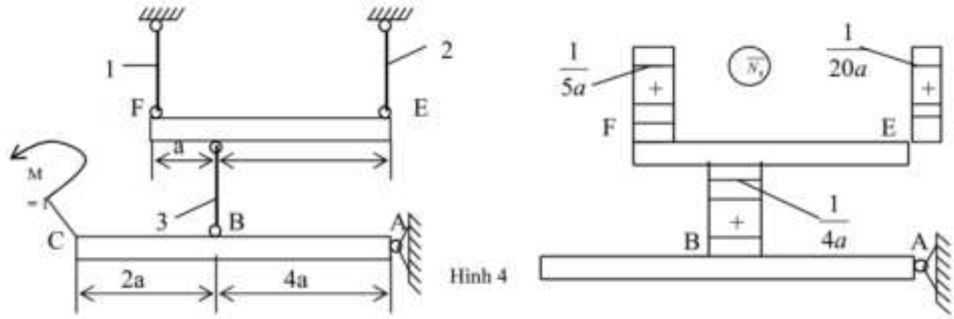

Fig-4: Internal diagram given for calculating rotational displacement for Example 1.

Discussion:
It is realized that geometrical method is also applied in this case to find the requirements. However, it is more convenient to apply graph multiplication method in this specific problem.

Example 2: Determine the internal forces for the following bars 1 and 2 (Figure 5). Known the bars have the same length l = a, cross section F, material (E).

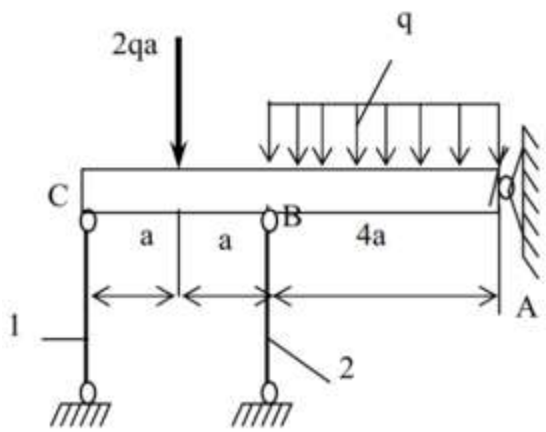

Fig- 5: Shamar for Example 2.

Solution
This problem is the statically indeterminate structure. Before doing some procedure, it should be changed the origin system become the equivalent system, namely statically determinate structure equivalent as presented in Figure 6.

The canonical equation is supported by the basic frame as presented in Figure 6.

$$\delta_{11}.X_1 + \Delta_{1P} = -\frac{X.a}{EF}$$

The right hand side of the equation is non-zero, since the displacement of point C is equal to the contraction of CD stick. Sign "-" because the displacement of C on AC is opposite to the direction of displacement of C on CD.

$$\delta_{11} = \frac{1}{EF}.a.\frac{3}{2}.\frac{3}{2} = \frac{9a}{4EF}, \ \Delta_{1P} = \frac{1}{EF}.\frac{9qa}{2}.a.\frac{3}{2} = \frac{27qa^2}{4EF}$$

$$\Leftrightarrow \frac{9qa}{4EF}.X_1 + \frac{27qa^2}{4EF} = -\frac{X_1 a}{EF} \Rightarrow X_1 = -\frac{27qa^2}{13}$$

The "-" sign indicates that the CD bar is under compression (reverse direction), this assume can be referred Figure 7.

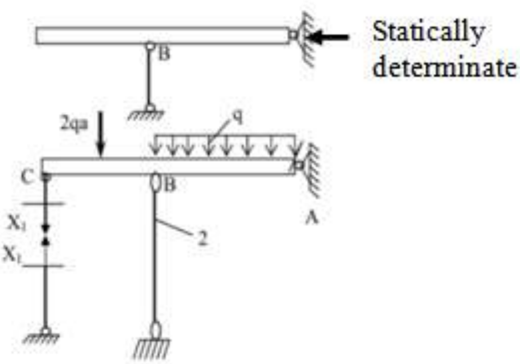

Fig- 6: Internal diagram given by both states for Example 2.

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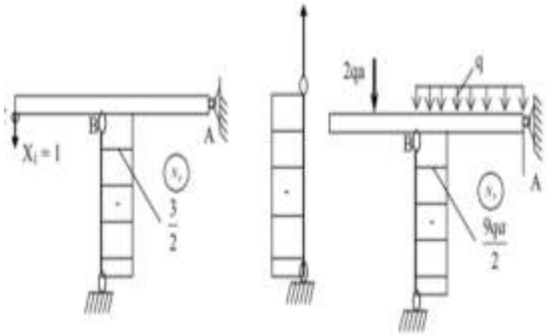

Fig- 7: Internal diagram given by both states for Example 2.

Discussion: In addition to the graph multiplication, the results in this problem can be found by other ones such as geometry, and the statically equivalent structure.

Example 3: Determine the vertical displacement at joint B of a load-bearing system as shown in Figure 8. The bar has a cross-sectional area F.

Solution

The longitudinal internal forces in the bars are:

$$N_1 = 2P.\cot g30^{\circ} = 2\sqrt{3}P$$ and

$$N_2 = \frac{P}{\sin 30^{\circ}} = 4P$$

The vertical force diagram due to $P_k = 1$ is shown in the figure. In the bars there is only longitudinal force, so we have:

$$y_B = \frac{1}{EF}[\Omega_1\eta_1 + \Omega_2\eta_2]$$

Where:

$$\Omega_1 = 2\sqrt{3}Pa.\cos 30^{\circ} = 3Pa : \eta_1 = \sqrt{3},\ \Omega_2 = 4Pa : \eta_2 = 2$$

Finally, the vertical displacement at B is

$$y_B = \frac{1}{EF}\left[3Pa.\sqrt{3} + 4Pa.2\right] = \frac{Pa}{EF}(8 + 3\sqrt{3})$$

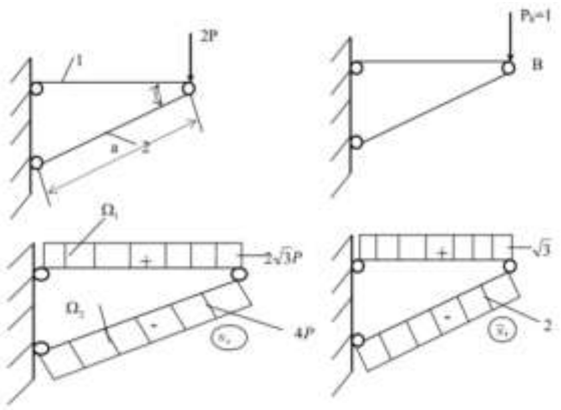

Fig- 8: Internal diagram given by both states for Example 3.

Discussion: In this case, if the current method is not used, it is complicated to utilized other method.

IV. CONCLUSIONS

This article presents the application of graph multiplication method to solve the rod system which has been not documented. Based on the origin expression of Maxwell-Mohr integration constructed to determining displacement and rotational displacement of any point in bending problem, the specific procedure applied to the rods which are only undergone axial tensile and compressive loading. The results revel that the graph multiplication method is completely used for these kinds of rob problems in both statically determinate and

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